

Applications



1. Consider rectangles with an area of 16 square inches.

a. Copy and complete the table.

Rectangles With Area 16 in.²

| | | | | | | | | |
|--------------|---|---|---|---|---|---|---|---|
| Length (in.) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Width (in.) | ■ | ■ | ■ | ■ | ■ | ■ | ■ | ■ |

b. Make a graph of the data.

c. Describe the pattern of change in width as length increases.

d. Write an equation that shows how the width w depends on the length ℓ . Is the relationship linear?

2. Consider rectangles with an area of 20 square inches.

a. Make a table of length and width data for at least five rectangles.

b. Make a graph of your data.

c. Write an equation that shows how the width w depends on the length ℓ . Is the relationship linear?

d. Compare and contrast the graphs in this exercise and those in Exercise 1.

e. Compare and contrast the equations in this exercise and those in Exercise 1.

3. A student collected these data from the bridge-length experiment.

Bridge-Length Experiment

| | | | | | |
|---------------------------|----|----|----|----|----|
| Length (in.) | 4 | 6 | 8 | 9 | 10 |
| Breaking Weight (pennies) | 24 | 16 | 13 | 11 | 9 |

a. Find an inverse variation equation that models the data.

b. Explain how your equation shows that breaking weight decreases as length increases. Is this decrease reasonable for the situation? Explain.

For Exercises 4–7, tell whether the relation between x and y is an inverse variation. If it is, write an equation for the relationship.

4.

| | | | | | | | | | | |
|-----|----|---|---|---|---|---|---|---|---|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

5.

| | | | | | | | | | | |
|-----|----|----|----|----|-----|---|-----|---|-----|-----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y | 48 | 24 | 16 | 12 | 9.6 | 8 | 6.8 | 6 | 5.3 | 4.8 |

6.

| | | | | | | | | | | |
|-----|----|----|----|------|----|-----|----|----|-----|-----|
| x | 2 | 3 | 5 | 8 | 10 | 15 | 20 | 25 | 30 | 40 |
| y | 50 | 33 | 20 | 12.5 | 10 | 6.7 | 5 | 4 | 3.3 | 2.5 |

7.

| | | | | | | | | | | |
|-----|-----|----|----|----|----|----|----|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| y | 100 | 81 | 64 | 49 | 36 | 25 | 16 | 9 | 4 | 1 |

8. The marathon is a 26.2-mile race. The best marathon runners can complete the race in a bit more than 2 hours.
- Make a table and graph that show how the average running speed for a marathon changes as the time to complete the race increases. Show times from 2 to 8 hours in one-hour intervals.
 - Write an equation for the relationship between time t and average running speed s for a marathon.
 - Tell how the average running speed changes as the time increases from 2 hours to 3 hours, from 3 hours to 4 hours, and from 4 hours to 5 hours.
 - How do the answers for part (c) show that the relationship between average running speed and time is not linear?

9. Testers drove eight vehicles 200 miles on a track at the same speed. The table below shows the amount of fuel each car used.

Fuel-Efficiency Test

| Vehicle Type | Fuel Used (gal) |
|-----------------|-----------------|
| Large Truck | 20 |
| Large SUV | 18 |
| Limousine | 16 |
| Large Sedan | 12 |
| Small Truck | 10 |
| Sports Car | 12 |
| Compact Car | 7 |
| Sub-Compact Car | 5 |



- Find the fuel efficiency in miles per gallon for each vehicle.
 - Make a graph of the (fuel used, miles per gallon) data. Describe the pattern of change shown in the graph.
 - Write a formula for calculating the fuel efficiency based on the fuel used for a 200-mile test drive.
 - Use your formula to find how fuel efficiency changes as the number of gallons of fuel increases from 5 to 10, from 10 to 15, and from 15 to 20.
 - How do the answers for part (d) show that the relationship between fuel used and fuel efficiency is not linear?
10. The route for one day of a charity bike ride covers 50 miles. Individual participants ride this distance at different average speeds.
- Make a table and a graph that show how the riding time changes as the average speed increases. Show speeds from 4 to 20 miles per hour in intervals of 4 miles per hour.
 - Write an equation for the relationship between the riding time t and average speed s .
 - Tell how the riding time changes as the average speed increases from 4 to 8 miles per hour, from 8 to 12 miles per hours, and from 12 to 16 miles per hour.
 - How do the answers for part (c) show that the relationship between average speed and time is not linear?

A number b is the **multiplicative inverse** of the number a if $ab = 1$. For example, $\frac{3}{2}$ is the multiplicative inverse of $\frac{2}{3}$ because $(\frac{3}{2} \cdot \frac{2}{3}) = 1$. For Exercises 22–27, find the multiplicative inverse of each number.

22. 2

23. -2

24. 0.5

25. 4

26. $\frac{3}{4}$

27. $\frac{5}{3}$

28. On a number line, graph each number in Exercises 22–27 and its multiplicative inverse. Describe any patterns you see.

Jamar takes a 10-point history quiz each week. Here are his scores on the first five quizzes: 8, 9, 6, 7, 10. Use this information for Exercises 29–30.

29. **Multiple Choice** What is Jamar's average quiz score?

A. 6

B. 7

C. 8

D. 9

30. Jamar misses the next quiz and gets a 0.

- What is his average after six quizzes?
- After 20 quizzes, Jamar's average is 8. He gets a 0 on the 21st quiz. What is his average after 21 quizzes?
- Why did a score of 0 have a different effect on the average when it was the sixth score than it did when it was the 21st score?

In Exercises 31 and 32, solve each equation using a symbolic method. Then describe how the solution can be found using a graph and a table.

31. $5x - 28 = -3$

32. $10 - 3x = 7x - 10$

For Exercises 33–35, find the equation of the line with the given properties.

33. slope $\frac{1}{2}$, y -intercept (0, 5)

34. slope 3, passing through the point (2, 2)

35. passing through the points (5, 2) and (1, 10)

36. Find the equation for the line shown below.



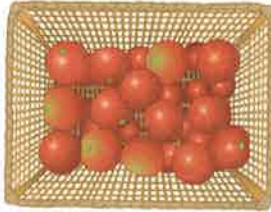
37. Suppose a car travels at a speed of 60 miles per hour. The function $d = 60t$ relates time t in hours and distance d in miles. This function is an example of *direct variation*. A relationship between variables x and y is a direct variation if it can be expressed as $y = kx$, where k is a constant.
- Describe two functions in this unit that are direct variations. Give the rule for each function as an equation.
 - For each function from part (a), find the ratio of the dependent variable to the independent variable. How is the ratio related to k in the general function?
 - Suppose the relationship between x and y is a direct variation. How do the y -values change as the x -values increase? How does this pattern of change appear in a graph of the relationship?
 - Compare direct variation and inverse variation. Be sure to discuss the graphs and equations of each.

For Exercises 38–40, tell which store offers the better buy. Explain your choice.

Gus's Groceries

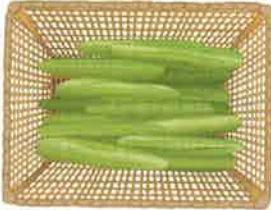
The Super Market

38. Tomatoes are
6 for \$4.00



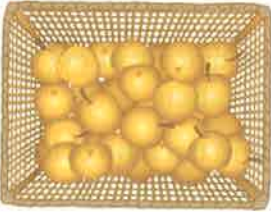
TOMATOES ARE
8 FOR \$4.60

39. Cucumbers are
4 for \$1.75



CUCUMBERS ARE
5 FOR \$2.00

40. Apples are
6 for \$3.00



APPLES ARE
5 FOR \$2.89

41. Suppose 6 cans of tomato juice cost \$3.20. Find the cost of the following numbers of cans.

a. 1 can

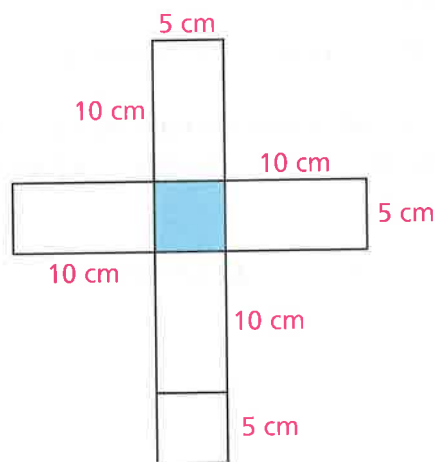
b. 10 cans

c. n cans



Extensions

- 42.** The drama club members at Henson Middle School are planning their spring show. They decide to charge \$4.50 per ticket. They estimate their expenses for the show at \$150.
- Write a function for the relationship between the number of tickets sold and the club's total profit.
 - Make a table to show how the profit changes as the ticket sales increase from 0 to 500 in intervals of 50.
 - Make a graph of the (tickets sold, total profit) data.
 - Add a column (or row) to your table to show the per-ticket profit for each number of tickets sold. For example, for 200 tickets, the total profit is \$750, so the per-ticket profit is $\$750 \div 200$, or \$3.75.
 - Make a graph of the (tickets sold, per-ticket profit) data.
 - How are the patterns of change for the (tickets sold, total profit) data and (tickets sold, per-ticket profit) data similar? How are they different? How are the similarities and differences shown in the tables and graphs of each function?
- 43.** The net below folds to make a rectangular prism.



- What is the volume of the prism?
- Suppose the dimensions of the shaded face of the prism are doubled. The other dimensions are adjusted so the volume remains the same. What are the new dimensions of the prism?
- Which prism has the smaller surface area, the original prism or the prism from part (b)? Explain.

44. Ms. Singh drives 40 miles to her sister's house. Her average speed is 20 miles per hour. On her way home, her average speed is 40 miles per hour. What is her average speed for the round trip?

For Exercises 45–47, find the value of c for which both ordered pairs satisfy the same inverse variation. Then write an equation for the relationship.

45. $(3, 16), (12, c)$

46. $(3, 9), (4, c)$

47. $(3, 4), (4, c)$

48. **Multiple Choice** The acceleration of a falling object is related to the object's mass and the force of gravity acting on it. For a fixed force F , the relationship between mass m and acceleration a is an inverse variation. Which equation describes the relationship of F , m , and a ?

A. $F = ma$

B. $m = Fa$

C. $\frac{m}{F} = a$

D. $\frac{m}{a} = F$

49. **Multiple Choice** Suppose the time t in the equation $d = rt$ is held constant. What happens to the distance d as the rate r increases?

F. d decreases.

G. d increases.

H. d stays constant.

J. There is not enough information.

50. **Multiple Choice** Suppose the distance d in the equation $d = rt$ is held constant. What happens to the time t as the rate r increases?

A. t decreases.

B. t increases.

C. t stays constant.

D. There is not enough information.



Mathematical Reflections

3

In this Investigation, you explored several examples of inverse variation and looked for patterns in the tables, graphs, and equations of these relationships. These questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **Suppose** the relationship between variables x and y is an inverse variation.
 - a. **How** do the values of y change as the values of x increase?
 - b. **Describe** the trend in a graph of (x, y) values.
 - c. **Describe** the equation that relates the values of x and y .
2. **How** is an inverse variation similar to a linear relationship? How is it different?



Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Nick described his thoughts in the following way:

In Question C of Problem 3.3, we looked at what happens to the cost per student when we double the number of students. Bill said that the per-student cost is cut in half. The cost per student when 20 students go is \$37.50. If 40 students go, the cost per student would be \$18.75.

When the number of students doubles from 40 to 80, the cost is cut in half again. This makes sense because you spread the cost among twice as many people, so each person pays half as much.

Then Jen asked, "What would happen if we tripled the number of students?" Bill explained that the cost per student would be one third as much. The cost per student when 20 students go is \$37.50, so the cost per student if 60 students go is one third of \$37.50, which is \$12.50.

Common Core Standards for Mathematical Practice

MP3 Construct viable arguments and critique the reasoning of others.



- What other Mathematical Practices can you identify in Nick's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.