Investigation 1

Exploring Data Patterns

People in many professions use data and mathematical reasoning to solve problems and make decisions. For example, engineers analyze data from lab tests to determine how much weight a bridge can hold. Market researchers use survey data to predict demand for new products. Stockbrokers use formulas to forecast growth of investments over time.

In several previous *Connected Mathematics* units, you used tables, graphs, and equations to explore and describe relationships between variables. In this Investigation, you will develop your skill in using these tools to organize data from experiments, find patterns, and make predictions.

1.1 Bridge Thickness and Strength

Many bridges are built with frames of steel beams. Steel is very strong, but any beam will bend or break if you put too much weight on it.

Common Core State Standards

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear) . . .

Also 8.F.A.2, 8.SP.A.1, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-BF.A.1a

- How do you think the strength of a beam is related to its thickness?
- What other variables might affect the strength of a bridge?

Problem 1.1

Engineers often use scale models to test their designs. You can do your own experiments to discover mathematical patterns involved in building bridges.

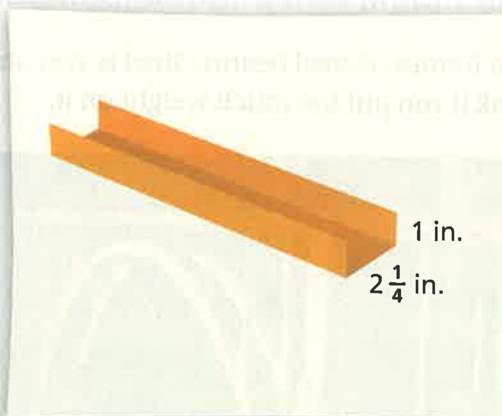
Instructions for a Bridge-Thickness Experiment

Materials:

- Two books of the same thickness
- A small paper cup
- About 50 pennies
- Several 11 inch-by-$4\frac{1}{4}$ inch strips of paper

Instructions:

- Start with one of the paper strips. Make a "bridge" by folding up 1 inch on each long side.



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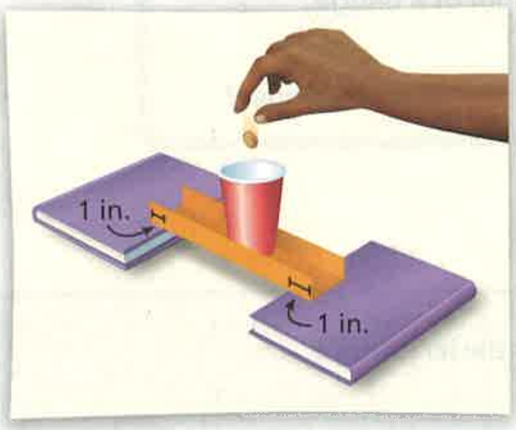
Problem 1.1 *continued*

- Suspend the bridge between the books. The bridge should overlap each book by 1 inch. Place the cup in the center of the bridge.
- Put pennies into the cup, one at a time, until the bridge collapses. Record the number of pennies you added to the cup. This number is the *breaking weight* of the bridge.



- Put two *new* strips of paper together to make a bridge with twice as many layers. Find the breaking weight for this bridge.
- Repeat this experiment to find the breaking weights of bridges made from three, four, and five strips of paper.

A Make a table and a graph of your (bridge layers, breaking weight) data.

B Does the relationship between the number of layers and the breaking weight seem to be linear or nonlinear? How do the graph and the table show this relationship?

C Suppose you could split layers of paper in half. What breaking weight would you predict for a bridge 2.5 layers thick? Explain.

D Predict the breaking weight for a bridge 6 layers thick. Explain your reasoning.

E Test your prediction of strength for the 6-layer bridge. Explain why results from such a test might not exactly match predictions.

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1.2 Bridge Length and Strength

In the last problem you tested the strength of some paper bridges. You found that bridges with more layers are stronger than bridges with fewer layers.

- How do you think the length and strength of a bridge are related?
- Are longer bridges stronger or weaker than shorter bridges?

Problem 1.2

You can do an experiment to find out how the length and strength of a bridge are related.

Instructions for a Bridge-Length Experiment

Materials:

- Two books of the same thickness
- A small paper cup
- About 50 pennies
- $4\frac{1}{4}$-inch-wide paper strips with lengths 4, 6, 8, 9, and 11 inches

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Problem 1.2 continued

Instructions:

- Fold the paper strips to make bridges as shown below.

- Start with the 4-inch bridge. Suspend the bridge between the two books as you did before. The bridge should overlap each book by 1 inch. Place the paper cup in the center of the bridge.
- Put pennies into the cup, one at a time, until the bridge collapses. Record the number of pennies you added to the cup. As in the first experiment, this number is the breaking weight of the bridge.
- Repeat the experiment to find breaking weights for the other bridges.

A Make a graph of your data.

B Describe the relationship between bridge length and breaking weight. How is that relationship shown by patterns in your table and graph?

C Use your data to predict the breaking weights for bridges of lengths 3, 5, 10, and 12 inches. Explain how you made your predictions.

D Compare your data from this experiment to the data from the experiment on bridges with different numbers of layers. How is the relationship between the number of layers in a bridge and its breaking weight similar to the relationship between bridge length and breaking weight? How is it different?

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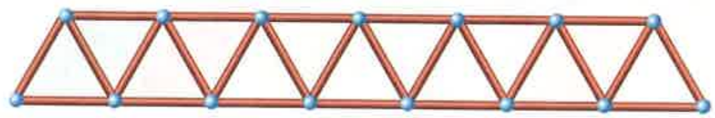
1.3 Custom Construction Parts
Finding Patterns

Suppose a company called Custom Steel Products (CSP for short) supplies materials to builders. One common structure that CSP makes is called a *truss*, as shown in the figure below. (You might see a truss holding up the roof of a building.) It is made by fastening together steel rods 1 foot long.

1-foot steel rod



7-foot truss made from 27 rods

This truss has an overall length of 7 feet. The manager at CSP needs to know the number of rods in any length of truss a customer might order.

Problem 1.3

Study the drawing above to see if you can figure out what the manager needs to know. It might help to work out several cases and look for a pattern.

A Copy and complete the table below to show the number of rods in trusses of different overall lengths.

Length of Truss (ft)	2	3	4	5	6	7	8
Number of Rods	7	11	■	■	■	27	■

1. Make a graph of the data in your table.
2. Describe the pattern of change in the number of rods used as the truss length increases.

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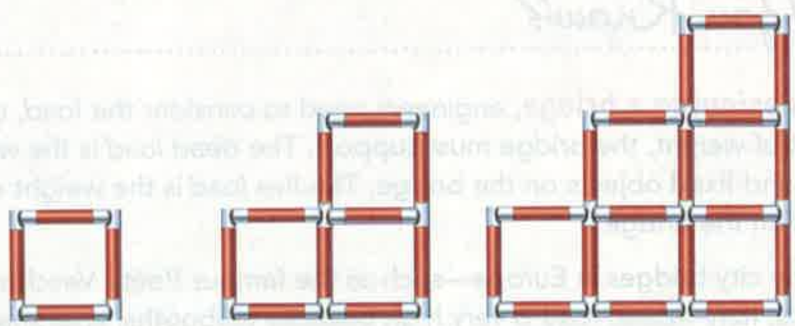
Problem 1.3 *continued*

3. How is the pattern you described shown in the table? How is it shown in the graph?
4. How many steel rods are in a truss 50 feet long overall? Explain how to find this number without drawing the truss.
5. By counting the triangles she could see for any length, Jenna says she figured out a pattern for the number of rods. For overall length 7, she sees 7 triangles and 6 rods connecting these triangles, so she writes $7 \times 3 + 6 = 27$. For length L, she writes $N = 3L + L - 1$. Explain where she gets the $3L$ and the $L - 1$ in her expression.

B Custom Steel Products also makes staircase frames like those shown here.



1 step made from 4 rods | 2 steps made from 10 rods | 3 steps made from 18 rods

1. Copy and complete the table below to show the number of rods in staircase frames with different numbers of steps.

CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	■	■	■	■	■

2. Make a graph of the data in your table.
3. Describe the pattern of change in the number of rods as the number of steps increases.
4. How is the pattern you described shown in the table? How is it shown in the graph?
5. How many steel rods are in a staircase frame with 12 steps? Explain how you could find this number without drawing the staircase frame.

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Problem 1.3 continued

C How is the pattern in Question A similar to the pattern in Question B? How is it different? Explain how the similarities and differences are shown in the tables and graphs.

D Compare the patterns of change in this problem with the patterns of change in Problems 1.1 and 1.2. Describe any similarities and differences you find.

ACE Homework starts on page 15.

Did You Know?

When designing a bridge, engineers need to consider the *load*, or the amount of weight, the bridge must support. The *dead load* is the weight of the bridge and fixed objects on the bridge. The *live load* is the weight of moving objects on the bridge.

On many city bridges in Europe—such as the famous Ponte Vecchio in Florence, Italy—dead load is very high because tollbooths, apartments, and shops are built right onto the bridge surface. Local ordinances can limit the amount of automobile and rail traffic on a bridge to help control live load.