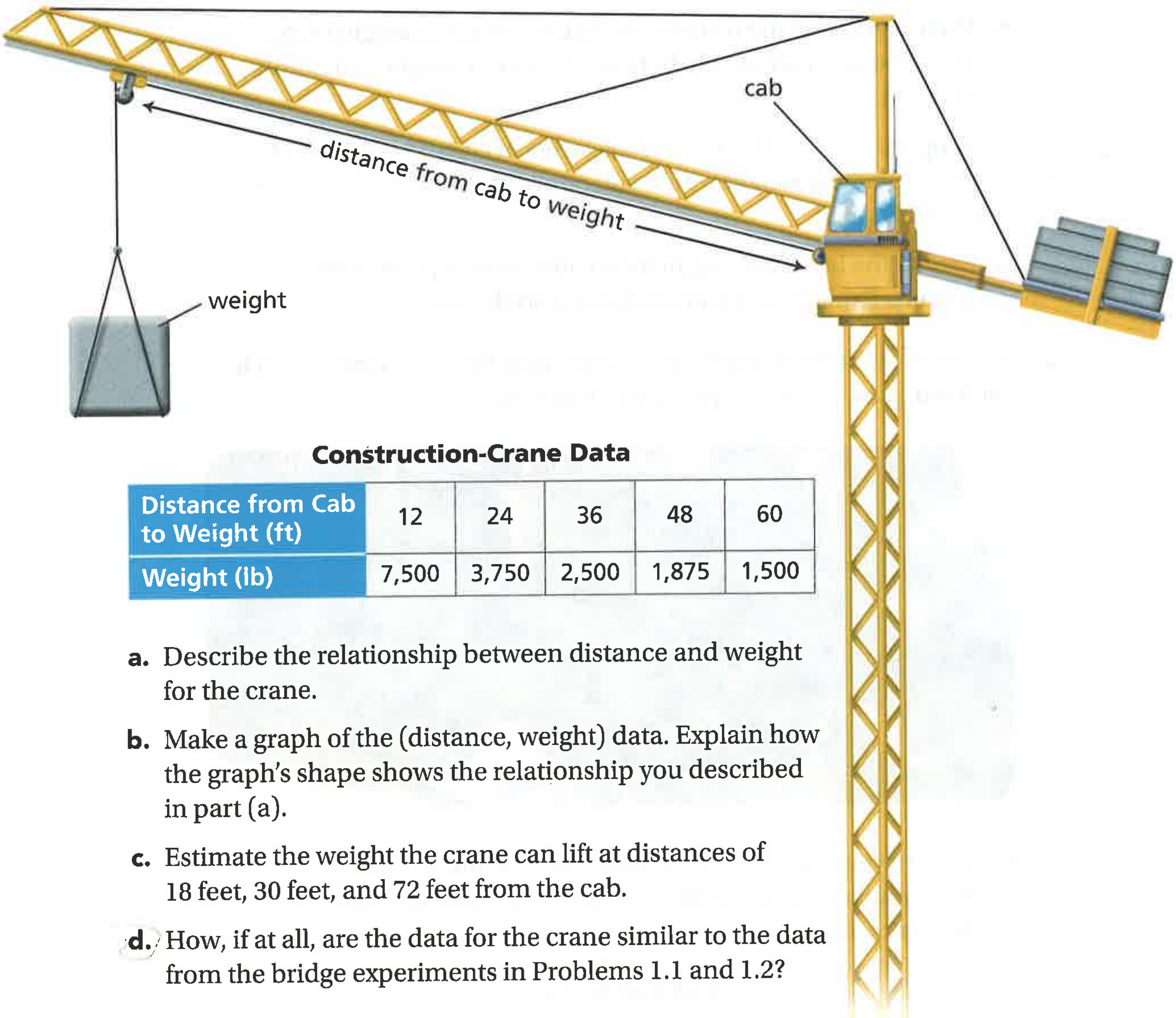
Applications | Connections | Extensions

Applications

1. The table shows the maximum weight a crane arm can lift at various distances from its cab.



Construction-Crane Data

Distance from Cab to Weight (ft)	12	24	36	48	60
Weight (lb)	7,500	3,750	2,500	1,875	1,500

a. Describe the relationship between distance and weight for the crane.

b. Make a graph of the (distance, weight) data. Explain how the graph's shape shows the relationship you described in part (a).

c. Estimate the weight the crane can lift at distances of 18 feet, 30 feet, and 72 feet from the cab.

d. How, if at all, are the data for the crane similar to the data from the bridge experiments in Problems 1.1 and 1.2?

2. A group of students conducted the bridge-thickness experiment with construction paper. The table below contains their results.

Bridge-Thickness Experiment

Number of Layers	1	2	3	4	5	6
Breaking Weight (pennies)	12	20	29	42	52	61

a. Make a graph of the (number of layers, breaking weight) data. Describe the relationship between breaking weight and number of layers.

b. Suppose it is possible to use half-layers of construction paper. What breaking weight would you predict for a bridge 3.5 layers thick? Explain.

c. Predict the breaking weight for a construction-paper bridge of 8 layers. Explain how you made your prediction.

3. A truss or staircase frame from Custom Steel Products costs $2.25 for each rod, plus $50 for shipping and handling.

a. Refer to your data from Question A of Problem 1.3. Copy and complete the table below to show the costs of trusses of different lengths.

Cost of CSP Truss

Truss Length (ft)	1	2	3	4	5	6	7	8
Number of Rods	3	7					27	
Cost of Truss								

b. Make a graph of the (truss length, cost) data.

c. Describe the relationship between truss length and cost.

d. Refer to your data from Question B of Problem 1.3. Copy and complete the table below to show the costs of staircase frames with different numbers of steps.

Cost of CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	■	■	■	■	■
Cost of Frame	■	■	■	■	■	■	■	■

e. Make a graph of the (number of steps, cost) data.

f. Describe the relationship between number of steps and cost.

4. During the medal ceremonies at a track meet, the top athletes stand on platforms made from stacked wooden boxes. The number of boxes depends on the number of medal winners.

1 medalist
1 box

2 medalists
3 boxes

3 medalists
6 boxes

a. Copy and complete the table below.

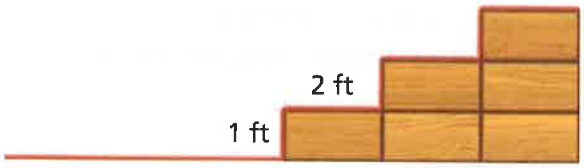
Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	■	■	■	■	■

b. Make a graph of the (number of medalists, number of boxes) data.

c. Describe the pattern of change shown in the table and graph.

d. Each box is 1 foot high and 2 feet wide. A red carpet starts 10 feet from the base of the platform and covers all the risers and steps.



Copy and complete the table below.

Carpet for Platforms

Number of Steps	1	2	3	4	5	6	7	8
Carpet Length (ft)	■	■	■	■	■	■	■	■

e. Make a graph of the (number of steps, carpet length) data.

f. Describe the pattern of change in the carpet length as the number of steps increases. Compare this pattern to the pattern in the (number of medalists, number of boxes) data.

5. Parts (a)–(f) refer to relationships between variables you have studied in this Investigation. Tell whether each is *linear* or *nonlinear*.

a. Cost depends on truss length (ACE Exercise 3).

b. Cost depends on the number of rods in a staircase frame (ACE Exercise 3).

c. Bridge strength depends on bridge thickness (Problem 1.1).

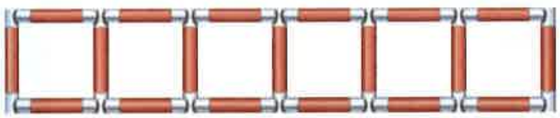
d. Bridge strength depends on bridge length (Problem 1.2).

e. Number of rods depends on truss length (Problem 1.3).

f. Number of rods depends on the number of steps in a staircase frame (Problem 1.3).

g. Compare the patterns of change for all the nonlinear relationships in parts (a)–(f).

6. CSP also sells ladder bridges made from 1-foot steel rods arranged to form a row of squares. Below is a sketch of a 6-foot ladder bridge.



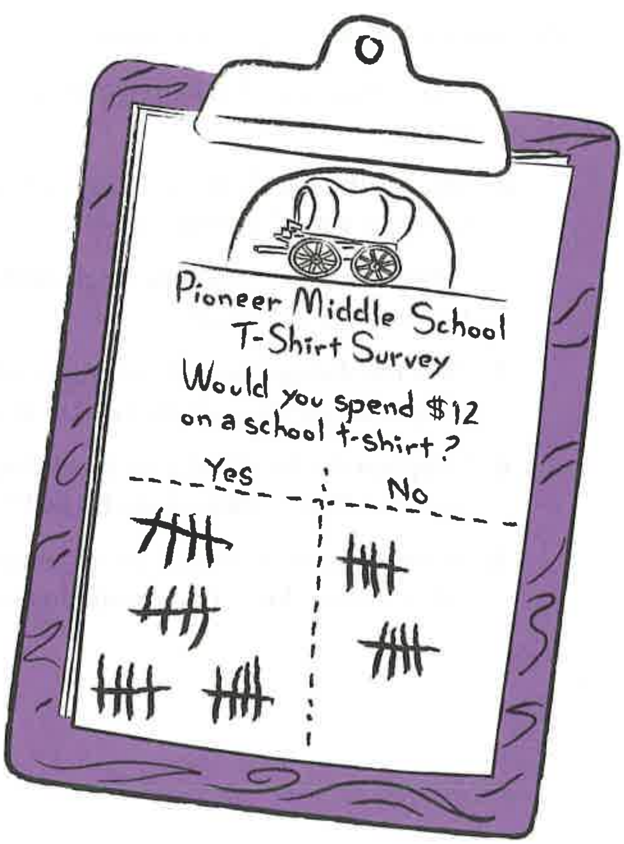
6-foot ladder bridge made from 19 rods

a. Make a table and a graph showing how the number of rods in a ladder bridge is related to the length of the bridge.

b. Compare the pattern of change for the ladder bridges with those for the trusses and staircase frames in Problem 1.3.

Connections

A survey of one class at Pioneer Middle School found that 20 out of 30 students would spend $12 for a school T-shirt. Use this information for Exercises 7 and 8.



7. **Multiple Choice** Suppose there are 600 students in the school. Based on the survey, how many students do you predict would spend $12 for a school T-shirt?

A. 20 **B.** 200

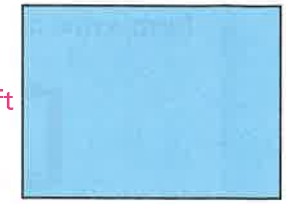
C. 300 **D.** 400

8. **Multiple Choice** Suppose there are 450 students in the school. Based on the survey, how many students do you predict would spend $12 for a school T-shirt?

F. 20 **G.** 200

H. 300 **J.** 400

9. At the right is a drawing of a rectangle with an area of 300 square feet.



   a. Make drawings of at least three other rectangles with an area of 300 square feet.

   b. What is the width of a rectangle with an area of 300 square feet if its length is 1 foot? If its length is 2 feet? If its length is 3 feet?

   c. What is the width of a rectangle with an area of 300 square feet and a length of ℓ feet?

   d. How does the width of a rectangle change if the length increases, but the area remains 300 square feet?

   e. Make a graph of (width, length) pairs for rectangles with an area of 300 square feet. Explain how your graph illustrates your answer for part (d).

10. The rectangle pictured in Exercise 9 has a perimeter of 70 feet.

   a. Make drawings of at least three other rectangles with a perimeter of 70 feet.

   b. What is the width of a rectangle with a perimeter of 70 feet if its length is 1 foot? 2 feet? ℓ feet?

   c. What is the width of a rectangle with a perimeter of 70 feet if its length is $\frac{1}{2}$ foot? $\frac{3}{2}$ feet?

   d. Give the dimensions of rectangles with a perimeter of 70 feet and length-to-width ratios of 3 to 4, 4 to 5, and 1 to 1.

   e. Suppose the length of a rectangle increases, but the perimeter remains 70 feet. How does the width change?

   f. Make a graph of (length, width) pairs that give a perimeter of 70 feet. How does your graph illustrate your answer for part (e)?

11. The 24 students in Ms. Cleary's homeroom are surveyed. They are asked which of several prices they would pay for a ticket to the school fashion show. The table shows the results.

Ticket-Price Survey

Ticket Price	$1.00	$1.50	$2.00	$2.50	$3.00	$3.50	$4.00	$4.50
Probable Sales	20	20	18	15	12	10	8	7

a. There are 480 students in the school. Use the data from Ms. Cleary's class to predict ticket sales for the entire school for each price.

b. Use your results from part (a). For each price, find the school's projected income from ticket sales.

c. Which price should the school charge if it wants to earn the maximum possible income?

12. At the right is a graph of the amount of money Jake earned while babysitting for several hours.

a. Put scales on the axes that make sense. Explain why you chose your scales.

b. What would the equation of the graph be, based on the scale you chose in part (a)?

c. If the line on this graph were steeper, what would it tell about the money Jake is making? Write an equation for such a line.

13. In each pair of equations below, solve the first and graph the second.

   a. $0 = 3x + 6$ $\quad$ $y = 3x + 6$

   b. $0 = x - 2$ $\quad$ $y = x - 2$

   c. $0 = 3x + 10$ $\quad$ $y = 3x + 10$

   d. In each pair, how is the solution related to the graph?

For Exercises 14–17, tell which graph matches the equation or the set of criteria.

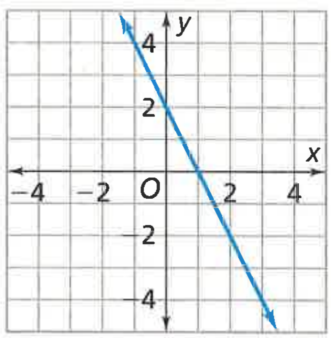
14. $y = 3x + 1$

15. $y = -2x + 2$

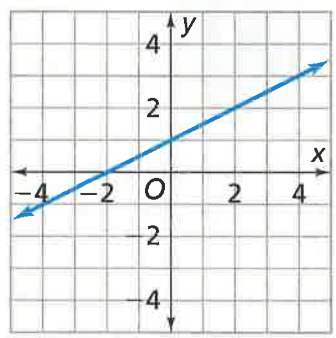
16. $y = x - 3$

17. y-intercept $= 1$; slope $= \frac{1}{2}$

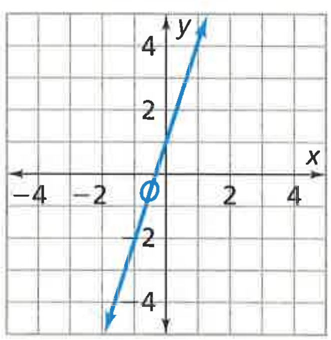
Graph A



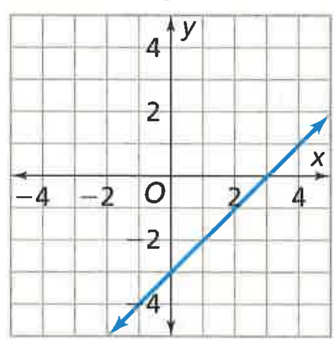
Graph B



Graph C



Graph D



In Exercises 18 and 19, each pouch holds the same number of coins. The coins all have the same value. Find the number of coins in each pouch. Explain your method.

18.

19.

20. Refer to Exercises 18 and 19.

a. For each exercise, write an equation to represent the situation. Let x represent the number of coins in a pouch.

b. Solve each equation. Explain the steps in your solutions.

c. Compare your strategies with those you used in Exercises 18 and 19.

In Exercises 21–28, solve each equation for x.

21. $3x + 4 = 10$

22. $6x + 3 = 4x + 11$

23. $6x - 3 = 11$

24. $-3x + 5 = 7$

25. $4x - \frac{1}{2} = 8$

26. $\frac{x}{2} - 4 = -5$

27. $3x + 3 = -2x - 12$

28. $\frac{x}{4} - 4 = \frac{3x}{4} - 6$

For Exercises 29–31, tell whether the statement is *true* or *false*. Explain your reasoning.

29. $6(12 - 5) > 50$

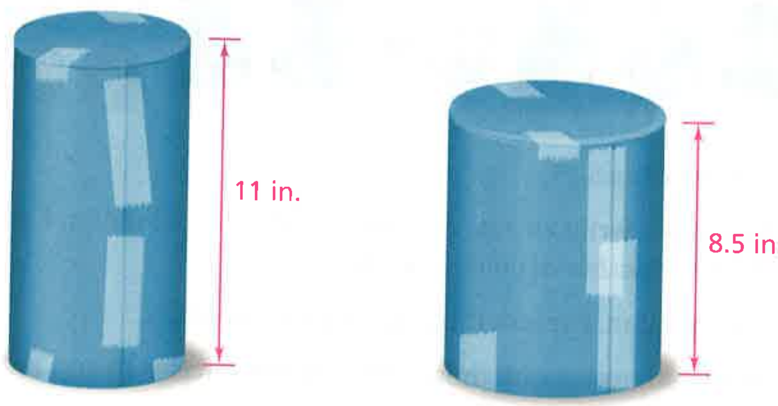
30. $3 \cdot 5 - 4 > 6$

31. $10 - 5 \cdot 4 > 0$

32. For this exercise, you will need two 8.5-inch by 11-inch sheets of paper and some scrap paper.

a. Roll one sheet of paper to make a cylinder 11 inches high. Overlap the edges very slightly and tape them together. Make bases for the cylinder by tracing the circles on the ends of the cylinder, cutting out the tracings, and taping them in place.

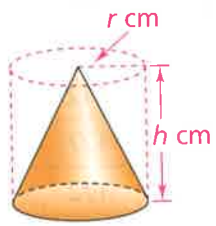
b. Roll the other sheet of paper to make a cylinder 8.5 inches high. Make bases as you did in part (a).



c. Do the cylinders appear to have the same surface area (including the bases)? If not, which has the greater surface area?

d. Suppose you start with two identical rectangular sheets of paper that are *not* 8.5 by 11 inches. You make two cylinders as you did before. Which cylinder will have the greater surface area, the taller cylinder or the shorter one? How do you know?

33. The volume of the cone in the drawing below is $\frac{1}{3}(28\pi)$ cm^3. Recall that the formula for the volume of a cone is $\frac{1}{3}\pi r^2 h$. What are some possible values of radius and height for the cone?



Extensions

34. Study the patterns in this table. Note that the numbers in the x column may not be consecutive after $x = 6$.

x	p	q	y	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	$\frac{1}{3}$
4	16	64	16	$\frac{1}{4}$
5	25	125	32	$\frac{1}{5}$
6	■	■	■	■
■	■	■	1,024	■
■	■	■	2,048	■
■	■	1,728	■	■
n	■	■	■	■

a. Use the patterns in the first several rows to find the missing values.

b. Are any of the patterns linear? Explain.

35. The table below gives data for a group of middle school students.

Data for Some Middle School Students

Student	Name Length	Height (cm)	Foot Length (cm)
Thomas Petes	11	126	23
Michelle Hughes	14	117	21
Shoshana White	13	112	17
Deborah Locke	12	127	21
Tonya Stewart	12	172	32
Richard Mudd	11	135	22
Tony Tung	8	130	20
Janice Vick	10	134	21
Bobby King	9	156	29
Kathleen Boylan	14	164	28

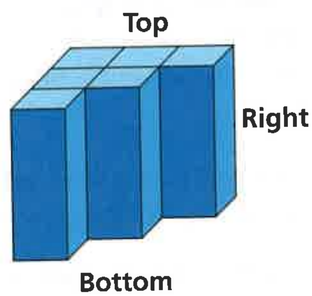
a. Make graphs of the (name length, height) data, the (name length, foot length) data, and the (height, foot length) data.

b. Look at the graphs you made in part (a). Which seem to show linear relationships? Explain.

c. Estimate the average height-to-foot-length ratio. How many foot-lengths tall is the typical student in the table?

d. Which student has the greatest height-to-foot-length ratio? Which student has the least height-to-foot-length ratio?

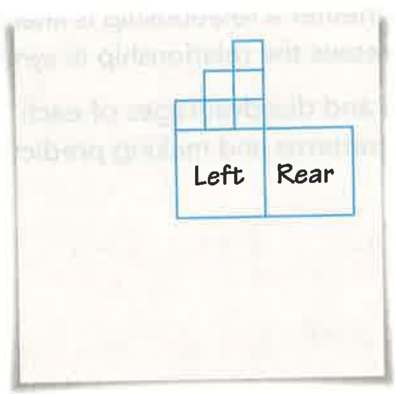
36. A staircase is a type of prism. This is easier to see if the staircase is viewed from a different perspective. In the prism shown here, each of the small squares on the top has an area of 1 square unit.



a. Sketch the base of the prism. What is the area of the base?

b. Rashid tries to draw a flat pattern that will fold up to form the staircase prism. Below is the start of his drawing. Finish Rashid's drawing and give the surface area of the entire staircase.

Hint: You may want to draw your pattern on grid paper and then cut it out and fold it to check.



c. Suppose the prism has six stairs instead of three. Assume each stair is the same width as those in the prism above. Is the surface area of this six-stair prism twice that of the three-stair prism? Explain.

Mathematical Reflections 1

In this Investigation, you used tables and graphs to represent relationships between variables and to make predictions. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. You can represent a relationship between variables with a table, a graph, a description in words, or an equation.
   a. How can you decide whether a relationship is linear by studying the pattern in a data table?
   b. How can you decide whether a relationship is linear by studying the pattern in a graph?
   c. How can you decide whether a relationship is linear by studying the words used to describe the variables?
   d. How can you decide whether a relationship is linear by studying the equation that expresses the relationship in symbolic form?
2. What are the advantages and disadvantages of each representation in finding patterns and making predictions?

Common Core Mathematical Practices

As you worked on the problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Jayden described his thoughts in the following way:

For Problem 1.1, we noticed from our table and graph that the data look linear. The graph shows this the best. The data points are in a nearly straight line.

In the table, the rate of change for different thicknesses varies from 7 pennies (change from 1 to 2 layers) to 10 pennies (change from 3 to 4 layers), with the average rate of change being about 8 pennies for each additional layer.

Some variability occurs because this is an experiment. We predict that 6 layers would hold about 50 pennies if we use the rate of change of 8 pennies to predict the increase.

Common Core Standards for Mathematical Practice

MP4 Model with mathematics

- What other Mathematical Practices can you identify in Jayden's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 2

Linear Models and Equations

In Investigation 1, you used tables, graphs, and equations to study relationships between variables. You found that the strength of a paper bridge depends on both its number of layers and its length. You found that the number of steel pieces needed to build a truss depends on the length of the truss. The number of pieces in a staircase frame depends on the number of steps.

If there is exactly one value of the dependent variable related to each value of the independent variable, mathematicians call the relationship a **function.** For example, the relationship between bridge breaking weight and length is a function. The relationship between the number of steel pieces used and the length of a truss is a function.

Common Core State Standards

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8a Understand that solutions to a system of two linear equations in two variables corresponds to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or with verbal descriptions).

Also **8.EE.C.8c, 8.F.A.3, 8.F.B.4, 8.F.B.5, 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, A-SSE.A.1a, A-CED.A.1, A-REI.C.6, F-IF.A.1, F-IF.C.7a, F-BF.A.1a, F-LE.A.2, F-LE.B.5, S-ID.B.6, S-ID.B.6a, S-ID.B.6b, S-ID.B.6c**