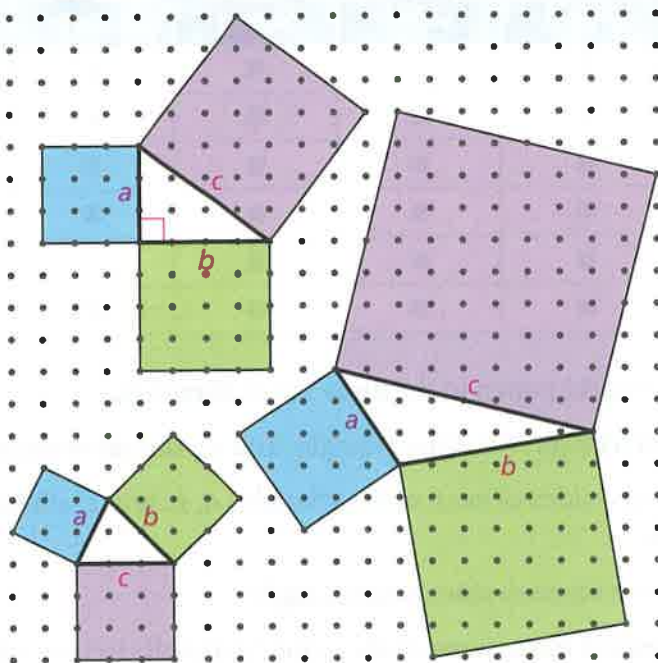


3.1 Discovering the Pythagorean Theorem

In this Investigation, you will use squares to find the side lengths of the acute, right, and obtuse triangles shown below.



- Is there a relationship among the areas of squares drawn on the sides of a triangle?
- If so, what is the relationship?
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Problem 3.1

- A** 1. Make a copy of the table below. Record the areas and lengths for the three given triangles.

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2. On dot paper or grid paper, make three more drawings.
- Draw a right triangle, an acute triangle, and an obtuse triangle.
 - Label the three sides of each of the triangles a , b , and c , where c the longest side.
 - Draw a square on each side of the triangles.
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- B** 1. For each triangle, look for a relationship among the areas of the three squares on the sides. Make a conjecture about the areas of the squares drawn on the sides of a triangle and the type of triangle.
2. Test your conjecture by drawing another triangle.

ACE Homework starts on page 49.

3.2 A Proof of the Pythagorean Theorem

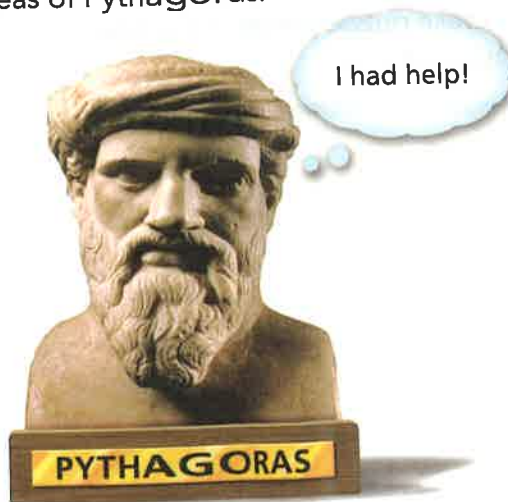
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More than 300 different proofs have been given for the Pythagorean Theorem. One of these proofs is based on the geometric argument you will explore in this Problem.

Did You Know?

Pythagoras lived in the 500s B.C. He had a devoted group of followers known as the Pythagoreans.

The Pythagoreans were a powerful group. Their influence became so strong that some people feared they threatened the local political structure. They were forced to disband. However, many Pythagoreans continued to meet in secret and to teach the ideas of Pythagoras.



The Pythagoreans held Pythagoras in **high** regard—so high that they gave him credit for all of their discoveries. **Much** of what we now attribute to Pythagoras may actually be the work of one or **several** of his followers. That includes the Pythagorean Theorem.



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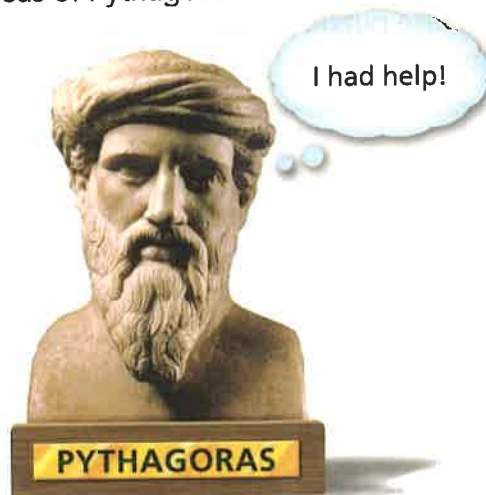
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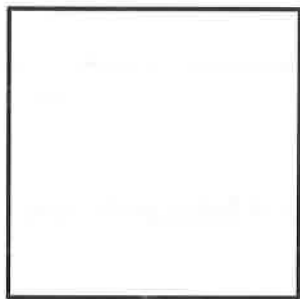
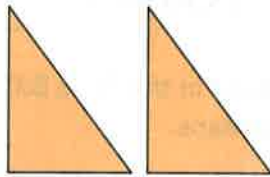
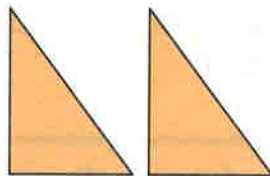
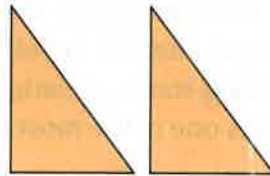
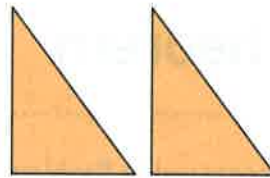
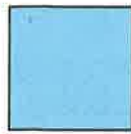
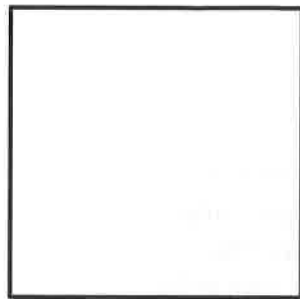
Problem 3.2

Copy the shapes on the previous page or use the puzzle pieces your teacher gives you.

- A** Study a triangle piece and the three square puzzle pieces. How do the side lengths of the squares compare to the side lengths of the triangle?
- B**
1. Arrange the 11 puzzle pieces to fit exactly into the two puzzle frames.
 2. What conclusion can you draw about the relationship among the areas of the three colored squares?
 3. What does the conclusion you reached in part (2) mean in terms of the side lengths of the triangles?
 4. Compare your results with those of another group. Did that group come to the same conclusion your group did? Is this conclusion true for all right triangles? Explain.
- C** Suppose a right triangle has legs of length 3 centimeters and 5 centimeters.
1. Use your conclusion from Question B to find the area of a square drawn on the hypotenuse of the triangle.
 2. What is the length of the hypotenuse?
- D** In Problem 3.1 and Problem 3.2, you have explored the Pythagorean Theorem, a relationship among the side lengths of a right triangle. State this theorem as a rule for any right triangle with leg lengths a and b and hypotenuse length c .

ACE Homework starts on page 49.

You can use the puzzle pieces below to explore the Pythagorean Theorem.



Puzzle frames

Puzzle pieces

? How can you use these puzzle pieces to prove the Pythagorean Theorem geometrically?

3.2 A Proof of the Pythagorean Theorem

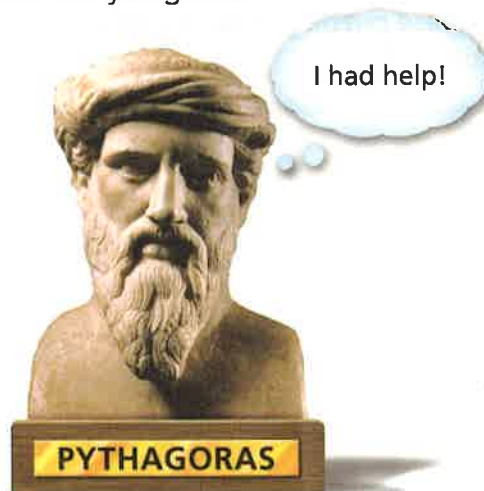
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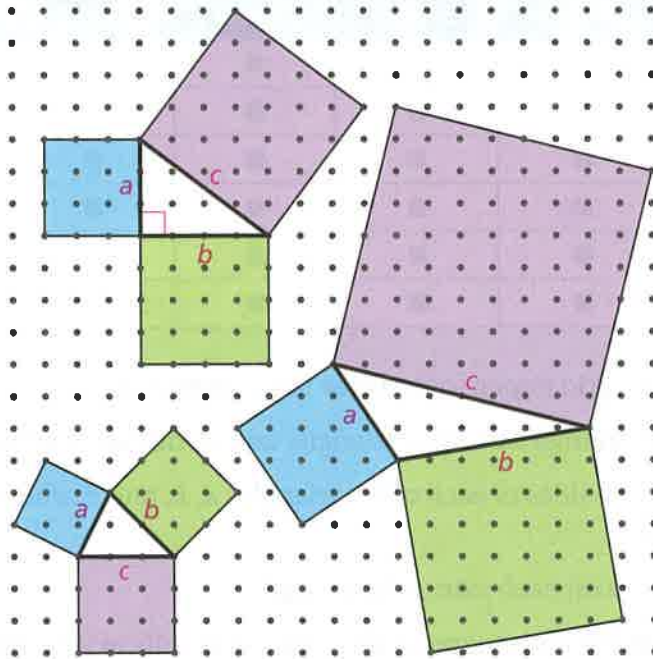


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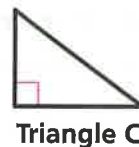
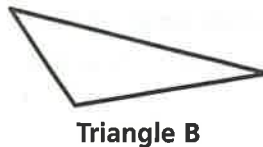


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The Pythagorean Theorem

In earlier grades, you learned about the properties of triangles. In this Investigation, you will learn about a special property of one type of triangle.

- What are characteristics that all triangles share?
- In what ways are the three triangles below different?
- How do the three side lengths of any triangle relate to each other?



Triangle A is an acute triangle. An **acute triangle** has three acute angles.

Triangle B is an obtuse triangle. An **obtuse triangle** has one obtuse angle.

Triangle C is a right triangle. A **right triangle** has one angle with a measure of exactly 90° . A 90° angle is called a *right angle* and is often marked with a small square. The longest side of a right triangle is the side opposite the right angle. This side is called the **hypotenuse**. The other two sides are called the **legs**.

- Can a triangle have more than one angle that is 90° ? Explain.

Common Core State Standards

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Ken described his thoughts in the following way:

In Problem 2.1, we showed that we had a square using the same reasoning we used in Problem 1.2 to show that the figures we drew on the coordinate grid were squares, rectangles, and right triangles. Then, to find the area of the square, we counted the number of square units inside the square.

Common Core Standards for Mathematical Practice

MP8 Look for and express regularity in repeated reasoning.



- What other Mathematical Practices can you identify in Ken's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.