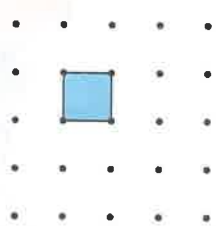
Investigation 2

Squaring Off

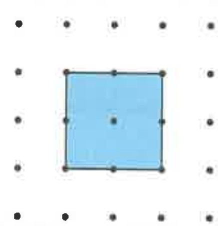
In this Investigation, you will explore the relationship between the side lengths and areas of squares. You will then use that relationship to find the lengths of segments on dot grids.

2.1 Looking for Squares

You can draw squares with different areas by connecting the points on a 5 dot-by-5 dot grid. Two simple examples follow.



area = 1 square unit



area = 4 square units

- What is the area of the largest square on a 5 dot-by-5 dot grid? Smallest square?

How many squares with different areas can you find?

Common Core State Standards

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram . . .

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes . . .

Also N-Q.A.3

Problem 2.1

A On 5 dot-by-5 dot grids, draw squares of various sizes by connecting dots. Draw squares with as many different areas as possible. Label each square with its area. Include at least two squares whose sides are not horizontal and vertical.

B Organize your set of squares by size. Then, describe the side lengths you found.

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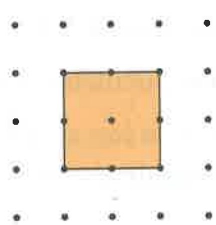
2.2 Square Roots

The area of a square is the length of a side multiplied by itself. This can be expressed by the formula $A = s \cdot s$, or $A = s^2$.

If you know the area of a square, you can work backward to find the length of a side. For example, suppose a square has an area of 4 square units. To find the length of a side, you need to figure out what positive number multiplied by itself equals 4. Because $2 \cdot 2 = 4$, the side length is 2 units. The number 2 is called a **square root** of 4.



This square has an area of 4 square units. The length of each side is the square root of 4 units, which is equal to 2 units.

In general, if $A = s^2$, then s is a square root of A. Because $2 \cdot 2 = 4$ and $-2 \cdot (-2) = 4$, 2 and -2 are both square roots of 4. Every positive number has two square roots. The number 0 has only one square root, 0.

For any positive number N, $\sqrt{N}$ indicates the positive square root of N. For example, $\sqrt{4} = 2$. The negative square root of 4 is $-\sqrt{4} = -2$.

- What is the side length of a square with an area of 2 square units?
- Is this length greater than 1? Is it greater than 2?
- Is 1.5 a good estimate for $\sqrt{2}$?
- Can you find a better estimate for $\sqrt{2}$?

Problem 2.2

In this Problem, use your calculator only when instructed to do so.

A 1. Find the side lengths of squares with areas of 1, 9, 16, and 25 square units.

2. Find the values of $\sqrt{1}$, $\sqrt{9}$, $\sqrt{16}$, and $\sqrt{25}$.

B 1. What is the area of a square with a side length of 12 units? What is the area of a square with a side length of 2.5 units?

2. Find the missing numbers.

   a. $\sqrt{\blacksquare} = 12$ b. $\sqrt{\blacksquare} = 2.5$

3. Find x.

   a. $x^2 = 121$ b. $x^2 = 2.25$

   c. $\sqrt{x} = 121$ d. $\sqrt{2.25} = x$

4. Explain what each positive value of x in part (3) might represent in terms of area and length.

C Refer to the square with an area of 2 square units you drew in Problem 2.1. The exact side length of this square is $\sqrt{2}$ units.

1. Estimate $\sqrt{2}$ by measuring a side of the square with a centimeter ruler.
2. Calculate the area of the square, using your measurement from part (1). Is the result exactly equal to 2? Could you use your ruler to make a more accurate measurement for $\sqrt{2}$? Explain.
3. Use the square root key on your calculator to estimate $\sqrt{2}$.
4. How does your ruler estimate compare to your calculator estimate?
5. Suppose you are designing a square sand box that has an area of 2 square meters. What is a reasonable and accurate measure for the side length?

Problem 2.2 continued

D 1. Between which two consecutive whole numbers does $\sqrt{5}$ lie? Explain.

2. Which whole number from part (1) is closer to $\sqrt{5}$? Explain.

3. Without using your calculator, estimate the value of $\sqrt{5}$ to one decimal place.

4. Without using your calculator, can you get an even closer estimate than in part (3)?

E Give the exact side length of each square you drew in Problem 2.1.

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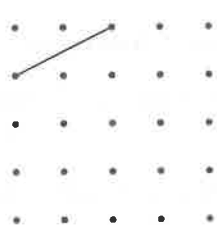
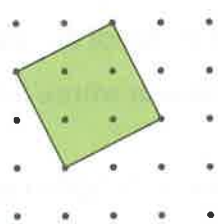
2.3 Using Squares to Find Lengths

You can use a square to find the length of a segment connecting dots on a grid. For example, to find the length of the segment on the left, draw a square with the segment as a side. The square has an area of 5 square units, so the segment has an exact length of $\sqrt{5}$ units.





- How can you find the exact length of a line segment connecting any two dots on grid paper?
- How many different length segments can you draw on the 5 dot-by-5 dot grid?

Problem 2.3

A 1. On 5 dot-by-5 dot grids, draw line segments with as many different lengths as possible by connecting dots. Label each segment with its length. Use the $\sqrt{\ }$ symbol to express lengths that are not whole numbers. (**Hint:** You will need to draw squares that extend beyond the 5 dot-by-5 dot grids.)

2. List the lengths in increasing order.

3. Estimate each nonwhole number length to one decimal place. Then locate the lengths on a number line. How can you use the number line to decide which length is the greatest? Least?

4. Describe a situation where measuring to one decimal place is not accurate enough.

B 1. Ella says the length of the segment in Figure 1 is $\sqrt{8}$ units. Oskar says it is $2\sqrt{2}$ units. Are both students correct? Explain.

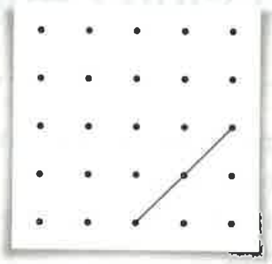


Figure 1

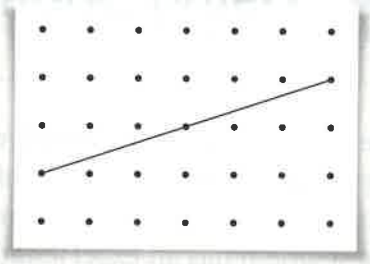


Figure 2

2. Express the exact length of the segment in Figure 2 in two ways.

3. Can you find a segment whose length cannot be expressed in two ways? Explain.

4. Which of the following lengths can be expressed in two ways: $\sqrt{5}$, $\sqrt{10}$, $\sqrt{18}$? Check your answers on a grid.

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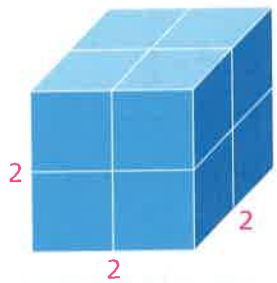
2.4 Cube Roots

The volume of a cube is the length of an *edge* multiplied by itself three times. Multiplying two edges of the base of a cube gives the area of the base. The area of the base times an edge that is the height gives the volume. The volume can be expressed by the formula $V = e \cdot e \cdot e$, or $V = e^3$.

If you know the volume of a cube, you can work backward to find the length of an edge. For example, suppose a cube has a volume of 8 cubic units. To find the length of an edge, you need to figure out what number multiplied by itself three times equals 8. Because $2 \cdot 2 \cdot 2 = 8$, the edge length is 2 units. The number 2 is called the **cube root** of 8.



This cube has the volume of 8 cubic units. The length of each edge is the cube root of 8 units, which is equal to 2 units.

In general, if $V = e^3$, then e is the cube root of V. Because $2 \cdot 2 \cdot 2 = 8$, 2 is the cube root of 8. Because $-2 \cdot (-2) \cdot (-2) = -8$, -2 is the cube root of -8.

You can use the symbol, $\sqrt[3]{\ }$, to indicate cube root. For any number N, $\sqrt[3]{N}$ indicates the cube root of N. For example, $\sqrt[3]{8} = 2$ and $\sqrt[3]{-8} = -2$.

- How is finding the cube roots the same or different from finding square roots?

Problem 2.4

In this Problem, use your calculator only when instructed to do so.

A

1. Find the edge lengths of cubes with volumes of 1, 27, 64, and 125 cubic units.
2. Find the values of $\sqrt[3]{1}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$, and $\sqrt[3]{125}$.

B

1. What is the volume of a cube with an edge length of 5 units? What is the volume of a cube with an edge length of 2.5 units?
2. Find the missing numbers.
   a. $\sqrt[3]{\blacksquare} = 5$
   b. $\sqrt[3]{\blacksquare} = 2.5$
3. Find x.
   a. $x^3 = 27$
   b. $x^3 = -27$
   c. $x^3 = \frac{1}{8}$
   d. $\sqrt[3]{x} = 27$
   e. $\sqrt[3]{x} = -27$
   f. $\sqrt[3]{x} = -\frac{1}{8}$
4. Explain what each positive value of x might represent in terms of volume and length.

C

1. Between which two consecutive whole numbers does $\sqrt[3]{10}$ lie? Explain.
2. Which whole number from part (1) is closer to $\sqrt[3]{10}$? Explain.
3. Without using your calculator, estimate the value of $\sqrt[3]{10}$ to one decimal place.
4. Without using your calculator, can you get an even closer estimate than in part (3)?
5. Three students find the edge length for a cube with a volume of 10 cubic inches. How might have each student arrived at their answer?

   Nick: 2.15 feet Josie: 2.1 feet Kevin: 2.1544 feet

D

1. Which is greater, $\sqrt{8}$ or $\sqrt[3]{8}$?
2. Which is greater, $\sqrt{N}$ or $\sqrt[3]{N}$? Explain.

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