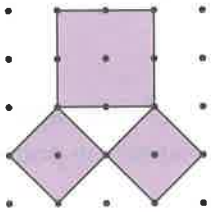
Applications

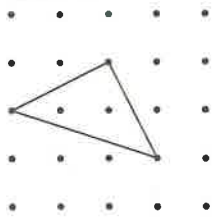
1. The diagram below shows a right triangle with a square on each side.



   a. Find the areas of the three squares.

   b. Use the areas from part (a) to show that the squares on the sides of this triangle satisfy the Pythagorean relationship, $a^2 + b^2 = c^2$.

2. The triangle below is a right triangle. Show that this triangle satisfies the Pythagorean Theorem.

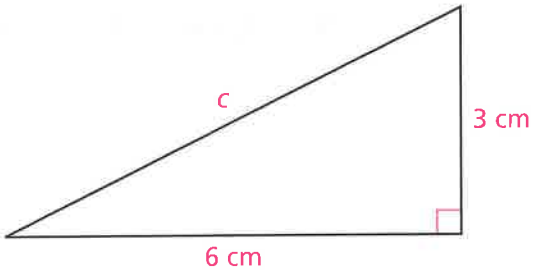


3. A right triangle has legs of length 5 inches and 12 inches.

   a. Find the area of a square drawn on the hypotenuse of the triangle.

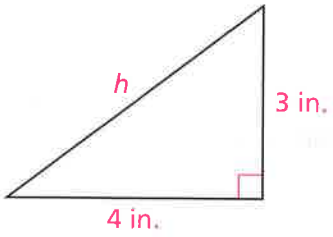
   b. Find the length of the hypotenuse.

4. Use the Pythagorean Theorem to find the length of the hypotenuse of this triangle.

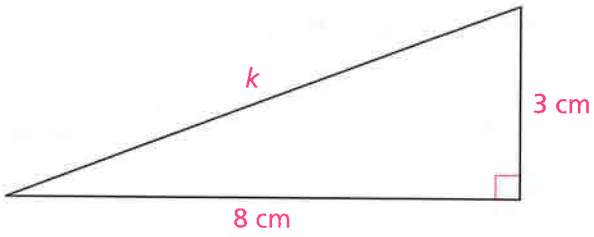


In Exercises 5 and 6, find each missing length.

5.



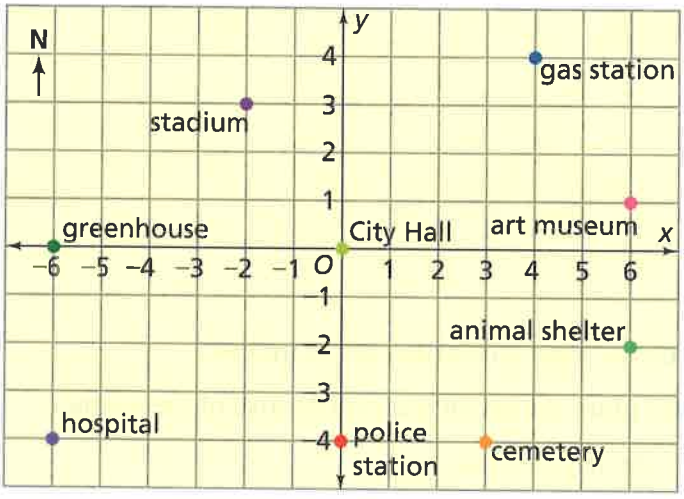
6.



7. On dot paper, find two points that are $\sqrt{17}$ units apart. Label the points W and X. Explain how you know the distance between the points is $\sqrt{17}$ units.

8. On dot paper, find two points that are $\sqrt{20}$ units apart. Label the points Y and Z. Explain how you know the distance between the points is $\sqrt{20}$ units.

For Exercises 9–12, use the map of Euclid. Find the flying distance in blocks between each pair of landmarks without using a ruler. Explain.



9. greenhouse and stadium

10. police station and art museum

11. greenhouse and hospital

12. City Hall and gas station

13. Multiple Choice Refer to the map of Euclid. Which landmarks are $\sqrt{40}$ blocks apart?

A. greenhouse and stadium

B. City Hall and gas station

C. hospital and art museum

D. animal shelter and police station

14. Multiple Choice Which set of side lengths makes a right triangle?

F. 10 cm, 24 cm, 26 cm

G. 4 cm, 6 cm, 10 cm

H. 5 cm, 10 cm, $\sqrt{50}$ cm

J. 8 cm, 9 cm, 15 cm

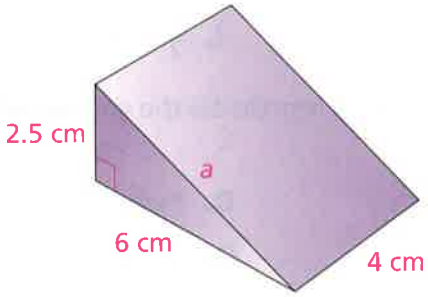
In Exercises 15 and 16, tell whether the triangle with the given side lengths is a right triangle.

15. 10 cm, 10 cm, $\sqrt{200}$ cm

16. 9 in., 16 in., 25 in.

Connections

17. The prism below has a base that is a right triangle.



a. What is the value of a?

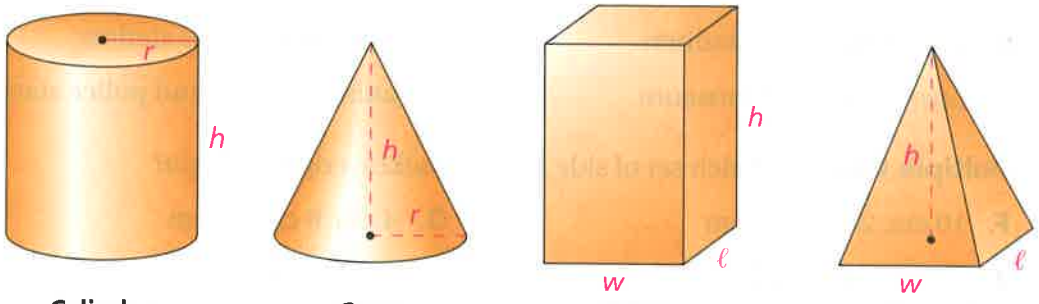
b. Do you need to know the value of a to find the volume of the prism? Do you need to know the value of a to find the surface area? Explain.

c. What is the volume?

d. What is the surface area?

e. Sketch a net for the prism.

For Exercises 18–21, refer to the figures below.



Cylinder Cone Prism Pyramid

18. Multiple Choice Which expression represents the volume of the cylinder?

A. $2\pi r^2 + 2\pi rh$ **B.** $\pi r^2 h$

C. $\frac{1}{3}\pi r^2 h$ **D.** $\frac{1}{2}\pi r^2 h$

19. Multiple Choice Which expression represents the volume of the cone?

F. $2\pi r^2 + 2\pi rh$ **G.** $\pi r^2 h$

H. $\frac{1}{3}\pi r^2 h$ **J.** $\frac{1}{2}\pi r^2 h$

20. Multiple Choice Which expression represents the volume of the prism?

A. $2(\ell w + \ell h + wh)$ **B.** ℓwh

C. $\frac{1}{3}\ell wh$ **D.** $\frac{1}{2}\ell wh$

21. Multiple Choice Which expression represents the volume of the pyramid?

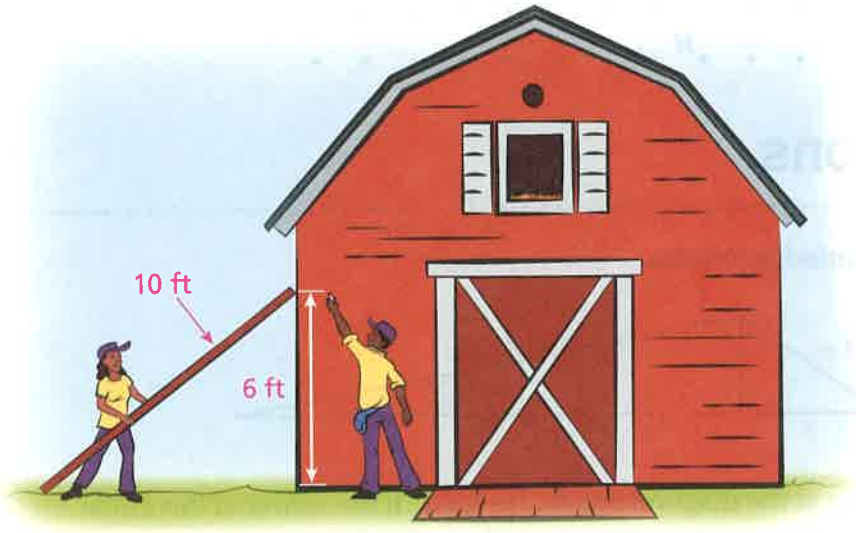
F. $2(\ell w + \ell h + wh)$ **G.** ℓwh

H. $\frac{1}{3}\ell wh$ **J.** $\frac{1}{2}\ell wh$

22. Nayo draws a quadrilateral. It has adjacent sides measuring 16 inches and 20 inches and a diagonal measuring 25 inches. Is her quadrilateral a rectangle? Explain.

23. Bo is building a tree house. He has marked locations for four holes that will hold his corner posts. They form a figure with a long side of 12 feet and a short side of 9 feet. What must the diagonal of the figure be to make sure the base of his tree house is a rectangle?

24. One method for checking whether a wall is perpendicular to the ground involves a 10-foot pole. A builder makes a mark exactly 6 feet high on the wall, and rests one end of the pole at that mark. The other end of the pole rests on the ground. A triangle is formed.



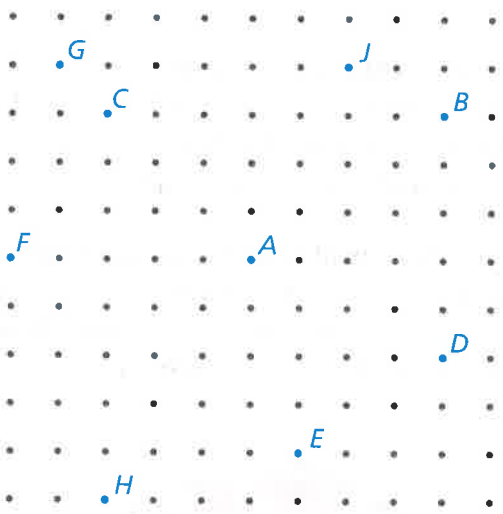
If the triangle is a right triangle, how far from the base of the wall is the bottom of the pole? Explain.

25. In the city of Euclid, Hilary's house is located at $(5, -3)$, and Jamilla's house is located at $(2, -4)$.

a. Without plotting points, find the shortest driving distance in blocks between the two houses.

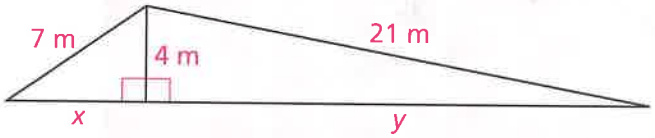
b. What is the exact flying distance between the two houses?

26. Which labeled point is the same distance from point A as point B is from point A? Explain.

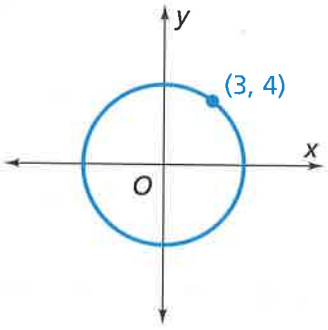


Extensions

27. Find the missing lengths.



28. Jolon looks at the diagram below. He says, "If the center of this circle is at the origin, then I can figure out the radius."

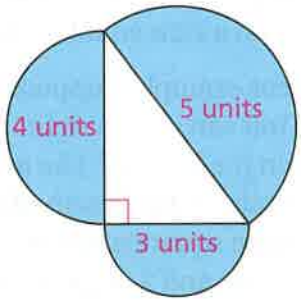


a. Explain how Jolon can find the radius.

b. What is the radius?

In Exercises 29–31, you will look for relationships among the areas of shapes other than squares drawn on the sides of a right triangle.

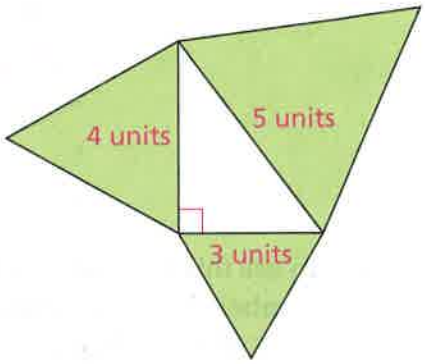
29. Half circles have been drawn on the sides of this right triangle.



a. Find the area of each half circle.

b. How are the areas of the half circles related?

30. Equilateral triangles have been drawn on the sides of this right triangle.



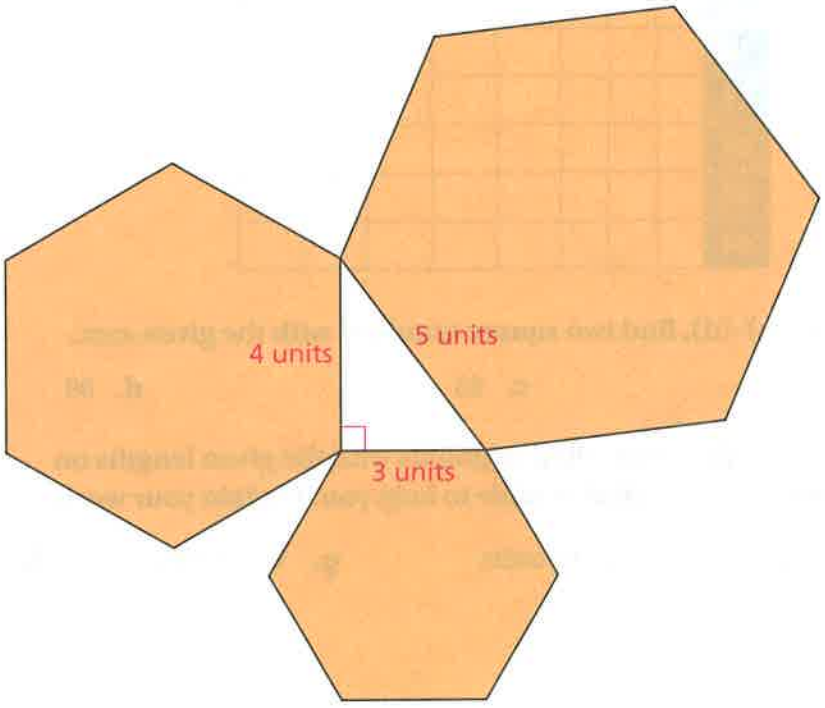
a. Find the area of each equilateral triangle.

b. How are the areas of the equilateral triangles related?

31. Regular hexagons have been drawn on the sides of this right triangle.

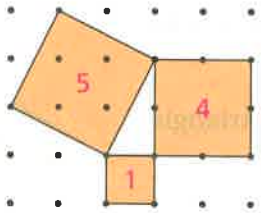
a. Find the area of each hexagon.

b. How are the areas of the hexagons related?



32. Any tilted segment that connects two dots on dot paper can be the hypotenuse of a right triangle. You can use this idea to draw segments of a given length. The key is finding two square numbers with a sum equal to the square of the length you want to draw.

For example, suppose you want to draw a segment with length $\sqrt{5}$ units. You can draw a right triangle in which the sum of the areas of the squares on the legs is 5. The area of the square on the hypotenuse will be 5 square units, so the length of the hypotenuse will be $\sqrt{5}$ units. Because 1 and 4 are square numbers, and $1 + 4 = 5$, a right triangle with legs of lengths 1 unit and 2 units has a hypotenuse of length $\sqrt{5}$ units.



a. To use this method, it helps to be familiar with sums of square numbers. Copy and complete the addition table to show the sums of pairs of square numbers.

+	1	4	9	16	25	36	49	64
1	2	5						
4	5							
9								
16								
25								
36								
49								
64								

For parts (b)–(d), find two square numbers with the given sum.

b. 10 **c.** 25 **d.** 89

For parts (e)–(h), draw tilted segments with the given lengths on dot paper. Use the addition table to help you. Explain your work.

e. $\sqrt{26}$ units **f.** 10 units **g.** $\sqrt{10}$ units **h.** $\sqrt{50}$ units

For Exercises 33–38, tell whether it is possible to draw a segment of the given length by connecting dots on dot paper. Explain.

33. $\sqrt{2}$ units

34. $\sqrt{3}$ units

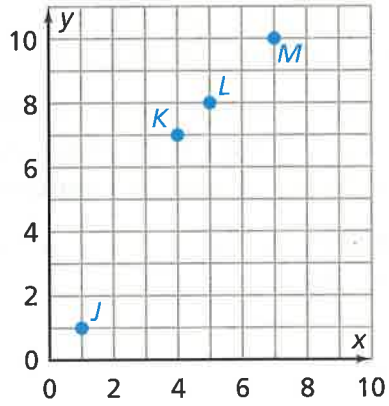
35. $\sqrt{4}$ units

36. $\sqrt{5}$ units

37. $\sqrt{6}$ units

38. $\sqrt{7}$ units

39. Use the graph to answer parts (a)–(c).



a. Find the coordinates of points J and K.

b. Use the coordinates to find the distance from point J to point K. Explain your method.

c. Use your method from part (b) to find the distance from point L to point M.

Mathematical Reflections 3

In this Investigation, you worked with a very important mathematical relationship called the Pythagorean Theorem. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Suppose you are given the lengths of two sides of a right triangle. **Describe** how you can find the length of the third side.
2. Suppose two points on a grid are not on the same horizontal or vertical line. **Describe** how you can use the Pythagorean Theorem to find the distance between the points without measuring.
3. **How** can you determine whether a triangle is a right triangle if you know only the lengths of its sides?