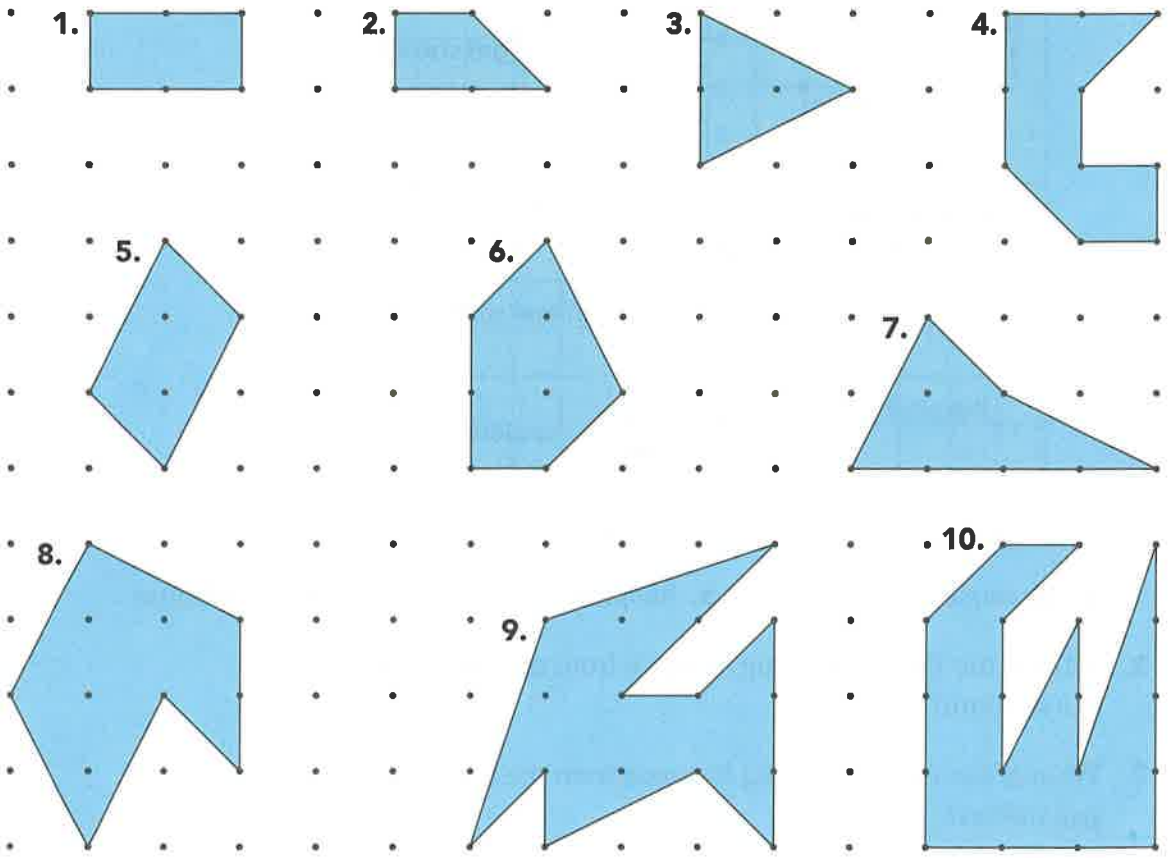
1.3 Finding Areas

Below are some park designs submitted to the Euclid City Council. To determine costs, the council needs to know the area of each park.



- How might you find the areas of irregular figures on dot paper?

Problem 1.3

Consider the horizontal or vertical distance between two adjacent dots to be 1 unit.

A Find the area of each figure.

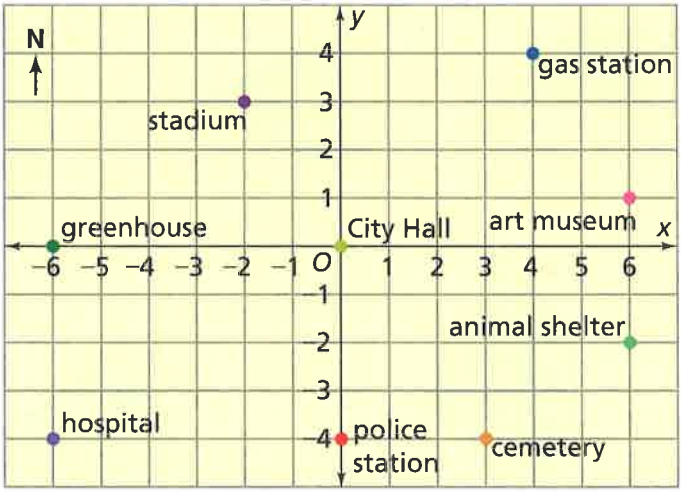
B Find the area of one of the square parks you suggested in Problem 1.2.

C Describe the strategies you used in Questions A and B.

ACE Homework starts on page 14.

Applications

For Exercises 1–7, use the map of Euclid from Problem 1.1.



1. Give the coordinates of each landmark.

   a. art museum **b.** hospital **c.** greenhouse

2. What is the shortest driving distance from the animal shelter to the stadium?

3. What is the shortest driving distance from the hospital to the gas station?

4. Suppose you travel by taxi. What are the coordinates of a point halfway from City Hall to the hospital? Is there more than one possibility? Explain.

5. Suppose you traveled by helicopter. What are the coordinates of a point halfway from City Hall to the hospital? Is there more than one possibility? Explain.

6. **a.** Which landmarks are 7 blocks from City Hall by car?

   b. Give precise driving directions from City Hall to each landmark you listed in part (a).

7. Euclid Middle School is located at the intersection of two streets. The school is the same driving distance from the gas station as the hospital is from the greenhouse.

a. List the coordinates of each place on the map where the school might be located.

b. Find the flying distance (in blocks) from the gas station to each location you listed in part (a).

The points (0, 0) and (3, 2) are two vertices of a polygon with integer coordinates.

8. Suppose the polygon is a square. What could the other two vertices be?

9. Suppose the polygon is a nonrectangular parallelogram. What could the other two vertices be?

10. Suppose the polygon is a right triangle. What could the other vertex be?

The points (3, 3) and (2, 6) are two vertices of a right triangle. Use this information for Exercises 11–13.

11. Multiple Choice Which point could be the third vertex of the right triangle?

A. (3, 2) **B.** (−1, 5) **C.** (7, 4) **D.** (0, 3)

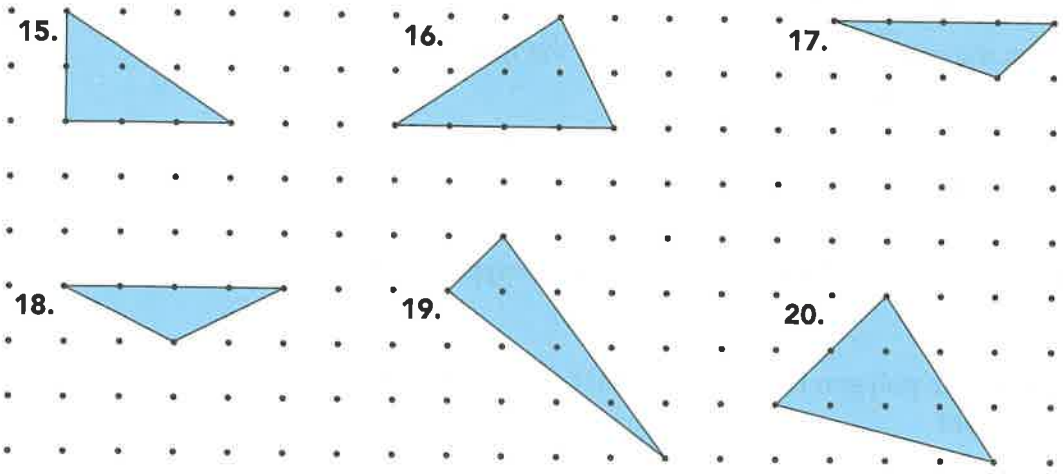
12. Give the coordinates of at least two other points that could be the third vertex.

13. How many right triangles with vertices (3, 3) and (2, 6) can you draw? Explain.

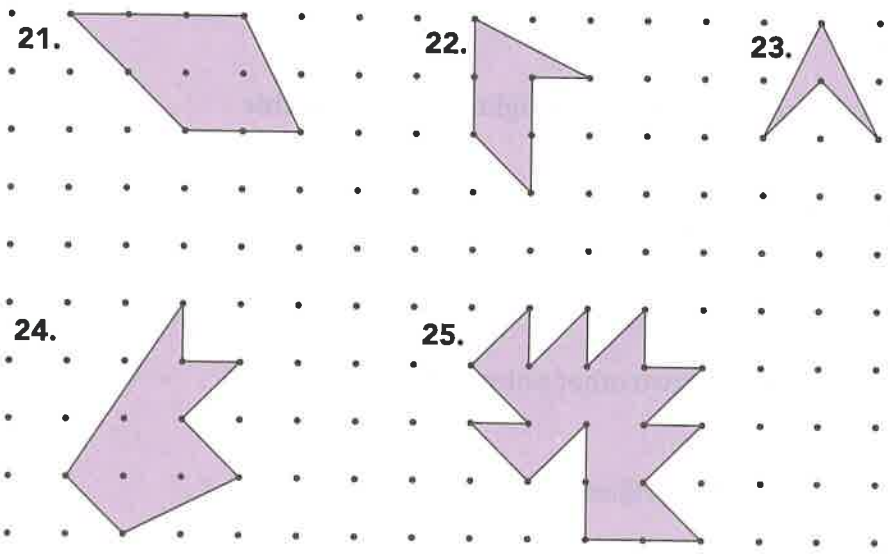
14. Can you connect the following points to form a parallelogram? Explain.

(1, 1) (2, −2) (4, 2) (3, 5)

Find the area of each triangle. If necessary, copy the triangles onto dot paper.

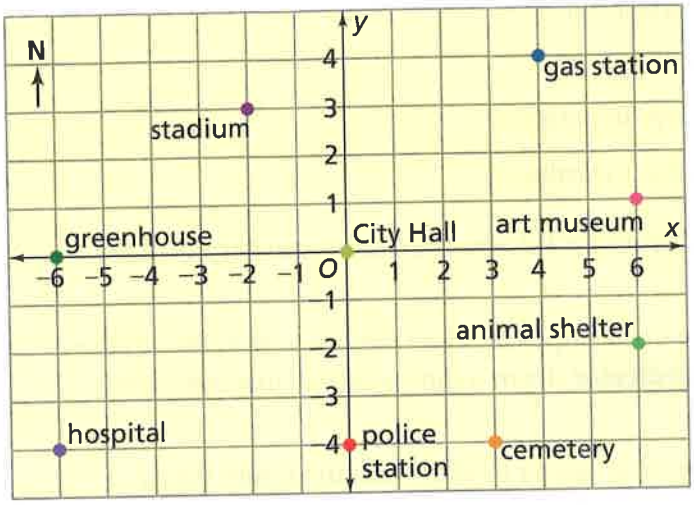


Find the area of each figure. Describe the method you use. If necessary, copy the figures onto dot paper.



Connections

In the city of Euclid, the length of each block is 150 meters. Use this information and the map from Problem 1.1 for Exercises 26–28.



26. What is the shortest driving distance, in meters, from City Hall to the animal shelter?

27. What is the shortest driving distance, in meters, from the police station to the gas station?

28. Between which two landmarks is the shortest driving distance 750 meters?

For Exercises 29–33, use the map of Euclid from Problem 1.1.

29. Matsu walks 2 blocks west from the police station and then walks 3 blocks north. Give the coordinates of the place where he stops.

30. Amy is at City Hall. She wants to meet Matsu at his ending location from Exercise 29. What is the shortest route she can take if she walks along city streets? Is there more than one possible shortest route?

31. Simon leaves the stadium and walks 3 blocks east, then 3 blocks south, then 2 blocks west, and finally 4 blocks north. Give the coordinates of the place where he stops.

32. Aida wants to meet Simon at his ending location from Exercise 31. She is at City Hall. What is the shortest route she can take if she walks along city streets? Is there more than one possible shortest route?

33. In general, how can you use coordinates to find the shortest walking route from City Hall to any point in Euclid?

34. Refer to the ordered pairs below. Do *not* plot the points on a grid to answer the questions. Explain each answer.

$(2, -3)$	$(3, -4)$	$(-4, -5)$	$(4, 5)$
$(-4, 6)$	$(-5, -5)$	$(0, -6)$	$(6, 0)$

a. Which point is farthest right?

b. Which point is farthest left?

c. Which point is above the others?

d. Which point is below the others?

35. When Fabiola solved Problem 1.2, she used slopes to help explain her answers.

a. In Question A, she used slopes to show that adjacent sides of the figure were **perpendicular** (form a right angle). How might she have done this?

b. In Question D, she used slopes to show that the figure was a parallelogram. How might she have done this?

36. Below are equations for eight lines.

line 1: $y = 3x + 5$	line 2: $y = 0.5x + 3$
line 3: $y = 10 - 2x$	line 4: $y = 1 - \frac{1}{3}x$
line 5: $y = 7 + 3x$	line 6: $y = -2x + 1$
line 7: $y = 5 + 6x$	line 8: $y = 3x$

a. Which of the lines are parallel to each other?

b. Which of the lines are perpendicular to each other?

37. Marcia finds the area of a figure on dot paper by dividing it into smaller shapes. She finds the area of each smaller shape and writes the sum of the areas as $\frac{1}{2} \cdot 3 + \frac{1}{2} + \frac{1}{2} + 1$.

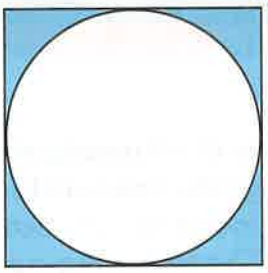
a. What is the total area of the figure?

b. On dot paper, draw a possible picture of the figure.

38. In the figure, a circle is inscribed in a square.

a. Find the area of the circle.

b. Find the area of the shaded region.



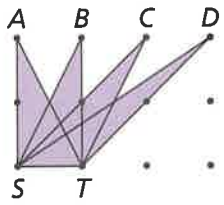
Extensions

39. Find a road map of your city or county. Figure out how to use the map's index to locate a city, street, or other landmark. How is finding a landmark by using an index description similar to and different from finding a landmark in Euclid by using its coordinates?

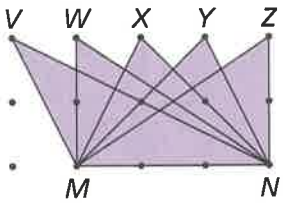
40. Use a map of your state to plan an airplane trip from your city or town to four other locations in your state. Write a set of directions for your trip that you could give to the pilot.

41. On grid paper, draw several parallelograms with diagonals that are perpendicular to each other. What do you observe about these parallelograms?

42. Find the areas of triangles *AST*, *BST*, *CST*, and *DST*. How do the areas compare? Why do you think this is true?



43. Find the areas of triangles *VMN*, *WMN*, *XMN*, *YMN*, and *ZMN*. How do the areas compare? Why do you think this is true?



Mathematical Reflections 1

In this Investigation, you solved problems involving coordinate grids. You located points, calculated distances and areas, and found the vertices of polygons that satisfied given conditions. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. In the city of Euclid, how does the driving distance from one place to another compare to the flying distance?
2. Suppose you know the coordinates of two landmarks in Euclid. How can you find the distance between the landmarks?
3. What are some strategies for finding areas of figures drawn on a grid?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Shawna described her thoughts in the following way:

We noticed that we could use a ruler to find the horizontal, vertical, or helicopter distance on the grid in Problem 1.1. Since each block was 1 centimeter, a ruler gave us another way to measure distance instead of counting blocks.

We also noticed that the diagonal of a square block is longer than each block, so counting blocks does not work for helicopter distances.

Common Core Standards for Mathematical Practice

MP5 Use appropriate tools strategically.

- What other Mathematical Practices can you identify in Shawna's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

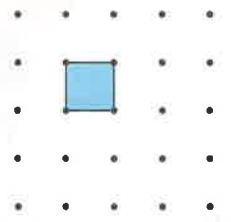
Investigation 2

Squaring Off

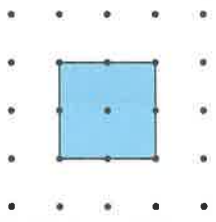
In this Investigation, you will explore the relationship between the side lengths and areas of squares. You will then use that relationship to find the lengths of segments on dot grids.

2.1 Looking for Squares

You can draw squares with different areas by connecting the points on a 5 dot-by-5 dot grid. Two simple examples follow.



area = 1 square unit



area = 4 square units

- What is the area of the largest square on a 5 dot-by-5 dot grid? Smallest square?

How many squares with different areas can you find?

Common Core State Standards

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram . . .

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes . . .

Also N-Q.A.3